VERSION Draft

Instructor:
Debashis Chatterjee
Assistant Professor
Dept. of Statistics
Siksha Bhavana

Visva Bharati

ODD SEMESTER , 2021
Campus: Santiniketan

STATISTICS

Mathematical Analysis
B.Sc 3rd semester
(Take Home Assignment 1)

(Time allowed: 2 hours)

NOTE: There are 10 questions. Each question carry 5 marks.

Sequence, Convergence of infinite series

1. Which of the following inequality is /are false? (write with explanation)

(a) $\liminf x_n + \limsup y_n \leq \limsup x_n + \limsup y_n$

(b) $\liminf x_n + \limsup y_n \leq \limsup (x_n + y_n)$

(c) $\liminf x_n + \limsup y_n \leq \limsup x_n + \limsup y_n$

(d) $\liminf x_n + \liminf y_n \leq \liminf x_n + \liminf y_n$

2. Which of the following inequality is /are true? (write with explanation)

(a) $\liminf x_n \leq \limsup x_n$

(b) $\liminf x_n \leq \lim x_n$

(c) $\limsup x_n \geq \lim x_n$

(d) $\liminf x_n \geq \limsup x_n$

3. Find supremum and infimum of x_n, where

(a) $x_n = (-50)^n$

(b) $x_n = \dfrac{n^2}{n^2 + 300}$

(c) $x_n = (-1)^n + \cos(\dfrac{n\pi}{2})$

(d) $x_n = (-1)^n + \cos(\dfrac{n\pi}{4})$

CONTINUED

Instructor:
Debashis Chatterjee
Assistant Professor
Dept. of Statistics
Siksha Bhavana

4. (write correct option with suitable explanation)
The sequence $\left\{\frac{n}{n+1}\right\}$ is

(a) increasing sequence

(b) decreasing sequence

(c) oscillatory

(d) unbounded sequence

5. Show the following (starting from definition of limit):
(Hint: Generally, we may use binomial expansion in power type sequence)

(a) Show that, $n^{1/n}$ converges to 1.

(b) Show that, $(n+1)^n$ converges to 1.

6. Let $a_n = \sqrt{n+1} - \sqrt{n}$ and $b_n = \sqrt{n^4+1} - n^2$. Then (write with explanation)

(a) $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} b_n$ diverges.

(b) $\sum_{n=1}^{\infty} a_n$ diverges but $\sum_{n=1}^{\infty} b_n$ converges.

(c) both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converges.

(d) both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges.

7. State Cauchy's 1^{st} limit theorem.
Using that, Show that $\lim_{n\to\infty} \frac{1}{n}\left[1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots + \frac{1}{n^2}\right] = 0.$

8. State Cauchy's 2^{nd} limit theorem.
Using that, Show that $\lim_{n\to\infty} \frac{(n!)^{1/n}}{n} = \frac{1}{e}.$

9. Show that, for $p, q > 0$, the series $\sum_{i=1}^{\infty} \frac{(i+1)^p}{i^q}$ is convergent for $p < q - 1$.

10. Show that, The series $\sum_{n=1}^{\infty} \frac{1}{\left(1+\frac{1}{n}\right)^{n^2}}$ is convergent.